DYNAMICS

For the Students of Undergraduate and Honours Classes and Various Competitive Examinations

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PREFACE

FIRST EDITION

This book has been designed to meet the requirements of undergraduate students of various Indian Universities. It is written in a simple class-room type language. The subject matter is presented in a lucid manner. Each article is followed by a variety of solved examples based on it, and a set of problems in the form of exercise. Most of the problems have been selected from university examination papers, so as to familiarize the students with the type of questions set in examination. Hints for many tricky problems are provided at respective places.

We hope that the book will cater to the needs of the students for whom it is meant. We are grateful to our colleagues for inspiring us to write this book.

Comments and suggestions to improve the book shall always be welcome.

K.K. Dixit
Vinod Kumar

THIRD EDITION

In this edition, the book has been revised and enlarged. Subject matter on Kepler's laws, motion of a particle on plane curves and motion of a particle in three dimensions has been added to fulfil the requirements of the new U.G.C. based syllabus.

We hope that the book in its present form, will be found more useful to the students.

K.K. Dixit
Vinod Kumar

CONTENTS

7. MOTION OF A PARTICLE IN THREE DIMENSIONS 244-263

Solution. Equations of a helix are

$$x = a\cos\theta, \quad y = a\sin\theta, \quad z = a\theta\tan\alpha$$

so that $\dfrac{dx}{dt} = -a\sin\theta\dfrac{d\theta}{dt}, \quad \dfrac{dy}{dt} = a\cos\theta\dfrac{d\theta}{dt}, \quad \dfrac{dz}{dt} = a\tan\alpha\dfrac{d\theta}{dt}.$

So, $$\left(\frac{ds}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2$$

$$= [a^2(\sin^2\theta + \cos^2\theta) + a^2\tan^2\alpha]\left(\frac{d\theta}{dt}\right)^2$$

$$= a^2\sec^2\alpha \cdot \left(\frac{d\theta}{dt}\right)^2,$$

where a is the radius and α the angle of helix.

Now if z_0 be the initial value of z, then the principle of energy gives $$2(z - z_0)g = \left(\frac{ds}{dt}\right)^2 = a^2\sec^2\alpha\left(\frac{d\theta}{dt}\right)^2,$$

or $$\left(\frac{d\theta}{dt}\right)^2 = \frac{2g}{a^2\sec^2\alpha}(z - z_0),$$ where $z = a\theta\tan\alpha$ and suppose that $z_0 = a\theta_0\tan\alpha$

$$= \frac{2ga\tan\alpha}{a^2\sec^2\alpha}(\theta - \theta_0) = \frac{2g\sin\alpha\cos\alpha}{a}(\theta - \theta_0)$$

$$\therefore \quad \sqrt{\left(\frac{2g\sin\alpha\cos\alpha}{a}\right)}\,dt = \frac{d\theta}{\sqrt{(\theta - \theta_0)}}.$$

Integrating, we get

$$t\sqrt{\left(\frac{2g\sin\alpha\cos\alpha}{a}\right)} = \int_{\theta_0}^{\theta_0 + 2\pi}\frac{d\theta}{\sqrt{(\theta - \theta_0)}} = 2\left[\sqrt{(\theta - \theta_0)}\right]_{\theta_0}^{\theta_0 + 2\pi}$$

$$= 2\sqrt{(2\pi)}.$$

Hence $$t = 2\sqrt{\left(\frac{\pi a}{g\sin\alpha\cos\alpha}\right)}.$$ ▲

EXERCISE 7.1

1. A particle is projected horizontally along the interior surface of a smooth hemisphere whose axis is vertical and whose vertex is downwards; the point of projection being at an angular distance β from the lowest point, show that the initial velocity so that the particle may just ascend to the rim of the hemisphere is $\sqrt{2ag\sec\beta}$.
2. A heavy particle is projected horizontally along the inner